Evolution of The Internal Model Principle: From Linear to Nonlinear

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Summary

The output regulation problem, or alternatively, the servomechanism problem, addresses the problem of designing a feedback controller to achieve asymptotic tracking for a class of reference inputs and disturbance rejection for a class of disturbances in an uncertain system while maintaining closed-loop stability. Thus, compared with the stabilization problem, the output regulation problem poses a more challenging problem because its formulation includes the stabilization problem as a special case. While the problem can be traced back as early as 1769 when James Watt devised fly-ball governors to regulate the steam engine speed in the presence of a variable load, the general formulation of this problem in a modern state-space framework was not given until the early 1970s. In contrast to some other trajectory tracking problems, where the trajectory to be tracked is assumed to be completely known, the output regulation problem aims to handle a class of reference inputs and a class of disturbances generated by differential equations. In what follows, the term "exogenous signals" will be used to refer to both reference inputs and disturbances when there is no need to distinguish them. The differential equation generating the exogenous signals will be called the exosystem.

The output regulation problem was first studied for the class of linear time-invariant systems under the assumption that the exosystem is an exactly known linear autonomous differential equation. It was completely solved by the collective efforts of many researchers, including Davison, Francis, and Wonham, to name just a few. A celebrated outcome of their research was what is called **The Internal Model Principle** which includes integral control as a special case. A feedback controller that solves the output regulator problem is called a regulator. A regulator is called structurally stable or robust if the regulator can tolerate small variations of the parameters of the plant. Wonham characterized the internal model principle as follows: **"A regulator is structurally stable only if the controller utilizes feedback of the regulated variable, and incorporates in the feedback loop a suitably reduplicated model of the dynamic structure of the exogenous signals which the regulator is required to process".** Wonham also summarized the internal model principle with the following plain terms: **"Every good regulator must incorporate a model of the outside world".**

At almost the same time that the solvability of the linear output regulation problem was completely understood, in the mid 1970s, Francis and Wonham also considered the output regulation problem for a class of nonlinear systems for the special case where the exogenous signals are constant. They showed that a linear regulator designed based on the linearized plant can solve the robust output regulation problem for a weakly nonlinear plant while maintaining the local stability of the closed-loop system. In the late 1980s, Huang and Rugh further studied this problem for general nonlinear systems using a gain scheduling approach, and their work related the solvability of this problem to the solvability of a set of nonlinear algebraic equations.

Since the 1990s, the research on the output regulation problem has been focused on nonlinear systems with time-varying exogenous signals. The foundation-laying work was given by Isidori and Byrnes in 1990 where they linked the solvability of the output regulation problem to a set of nonlinear partial differential and algebraic equations called regulator equations. Based on the solution of the regulator equations, both state feedback and error feedback controllers can be synthesized to solve the output regulation problem for an exactly known plant while maintaining the local stability of the closed-loop system. The regulator presented by Isidori and Byrnes is a combination of feedback control and feedforward control. Nevertheless, such a control strategy is not structurally stable. To deal with the output regulation problem with uncertain nonlinear system, a natural approach is to apply a regulator incorporating an internal model in the sense of Wonham. Unfortunately, this idea does not work as shown by a counterexample by Byrnes and Isidori. Huang realized in 1991 that the reason for the failure of the linear internal model principle for nonlinear output regulation problem is that, unlike the linear case, the steady-state tracking error in a nonlinear system is a nonlinear function of the exogenous signals, and, a regulator incorporating a (linear) exosystem can only robustly annihilates the linear term of the steady-state tracking error. **Thus, for nonlinear systems, a regulator incorporating a model of the outside world is not good enough.** Based on this observation, Huang found that if the solution of the regulator equations is a polynomial in the exogenous signals, then it is possible to synthesize a structurally stable regulator for uncertain nonlinear systems by utilizing the so-called k-fold internal model. This approach effectively leads to a nonlinear version of the internal model principle. Subsequently, the robust output regulation problem was further pursued by Byrnes and Isidori, Delli Priscoli, and Khalil, generating various techniques and insights on this important issue. In particular, the local robust output regulation problem has been extended to regional and global versions by Huang and Chen, Khalil, and (Isidori, Serrani and Isidori). More recently, the formulation of the output regulation problem has been generalized to the case where the exosystem is uncertain. This case necessitates the employment of adaptive control techniques, thus stimulating a new round of research activities. See- Ding, (Liu, Chen, and Huang), Marino and Tomei, Nikiforov, (Serrani, Marconi and Isidori), and Ye and Huang. It is noted that this scenario had not been studied previously even for linear systems.

As we have seen, the scope of research on the output regulation problem has been constantly expanding, and this research field is being made richer and more interesting with the injections of new ideas and techniques from other research areas such as robust control, adaptive control, optimal control, neural networks, and numerical mathematics. Due to this expansion, the concept of the internal model needs to be examined, modified, and extended. Indeed, the effort for establishing a nonlinear version of internal model started in early 1990s and the concept of internal model has been in the process of constant evolution.

For linear systems, an internal model is defined by a pair of controllable matrices and can be constructed readily by using the knowledge of the minimal polynomial of the exosystem. Clearly, this type of definition cannot be generalized to define a nonlinear internal model. Nevertheless, from the role of the internal model, we can also define an internal model as a dynamic compensator. Attaching the internal model to the given plant yields a so-called augmented system with two properties: the augmented system is stabilizable, and the stabilization solution of the augmented system leads to the solution of the output regulation problem of the given plant. With this concept of internal model, we can see that the key to solving the linear output regulation problem is to find an internal model, i.e., to find a dynamic compensator with the aforementioned two properties.

One may immediately question whether or not such a nicely-defined creature exists at all! This question is legitimate in the sense that the problem of global stabilizability for a general nonlinear system is already untractable. Acknowledging this fact, we further introduce the concept of the internal model candidate which is a dynamic compensator with the second property of the internal model. In other words, existence of an internal model candidate implies that the output regulation problem of the given plant can be converted into a stabilization problem of the augmented system. It can be shown that the existence of an internal model candidate boils down to the existence of an autonomous system that is able to reproduce the solution of the regulator equations. Since the solution of the regulator equations reflects the steady-state behavior of the plant and the exosystem, we call such an autonomous system a steady-state generator of the plant and the exosystem. It turns out that, in many interesting cases, a steady-state generator can be constructed based on the solution of the regulator equations. The first case is when the solution of the regulator equations is a polynomial in the exogenous signal given by Huang. The second case is when the solution of the regulator equations is an arbitrarily known (nonlinear) function of some polynomials in the exogenous signal given by Chen and Huang. The second case obviously contains the first case as a special case. The third case is when the solution of the regulator equations satisfies a nonlinear autonomous differential equation given by Byrnes and Isidori, and Delli Priscoli. This case also includes the first case as a special case since, in the first case, the solution of the regulator equations satisfies a linear autonomous differential equation. While all the above three cases make use of the notion of system immersion, a nonlinear observer based internal model was recently proposed for the class of output feedback systems (Marconi, Praly, and Isidori).

The concept of the steady-state generator not only enables the conversion of the output regulation problem of the given plant to the stabilization problem of the augmented system, but also leads to the construction of some prototypes of the internal model by viewing an internal model as some type of steady-state observer of the steady-state generator. In particular, when the solution of the regulator equations is polynomial in the exogenous signal, there exists a canonical linear internal model which is independent of the uncertain parameter of the exosystem, thus enabling the solvability of this case by adaptive control techniques.

This talk will be centered on the evolution of the concept of internal model and will be divided into the following parts. Part 1 is an introduction to the output regulation problem and the internal model design. In Part 2, we give a new characterization of the internal model as follows. First, we define an internal model candidate as any dynamic system which together with the given plant constitutes a so-called augmented system with the property that the stabilizability of the augmented system implies the solvability of the output regulation problem of the given plant. An internal model candidate is further called an internal model if it is such that the augmented system is stabilizable. In Part 3, we show some scenarios where a special type of internal model candidate called steady-state generator can be constructed which in turn leads to the solvability of the global robust output regulation problem for some typical nonlinear systems such as output feedback systems, strictly-feedback systems, and strictly-feedforward systems. Part 4 presents some applications of the internal model design including the synchronization of a harmonic system with the FitzHugh-Nagumo model which is a chaotic system, disturbance rejection of the RTAC system which is also known as a nonlinear benchmark control problem, and the adaptive attitude tracking and disturbance rejection of a spacecraft system. Finally, we close the talk by summarizing the extension of the internal model principle from linear domain to nonlinear domain as follows:

A regulator is structurally stable only if the controller utilizes feedback of the regulated variable, and incorporates in the feedback loop a steady-state generator of the plant and the exosystem.

In short, **every good regulator must incorporate a steady-state generator.**

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